Solve the following differential equations. SCORE: _____ / 30 PTS

[a] $y\,dy + (y^2 + xy^4)dx = 0$ FINAL ANSWER: $y^{-2} = -x - \frac{1}{2} + Ce^{2x}$

13 POINTS

[b] $2r\,dr + (1 - r^2 \tan\theta)d\theta = 0$ FINAL ANSWER: $r^2\cos\theta + \sin\theta = C$

10 POINTS

[c] $\dfrac{dx}{dt} = \dfrac{x(x^2 - 3t^2)}{t(x^2 - t^2)}$ FINAL ANSWER: $t^6 e^{\frac{x^2}{t^2}} = Cx^2$

8 POINTS

[a] $\dfrac{dy}{dx} + y = -xy^3$ BERNOULLI

$v = y^{1-3} = y^{-2}$

$\dfrac{dv}{dx} = -2y^{-3}\dfrac{dy}{dx}$

$-2y^{-3}\dfrac{dy}{dx} - 2y^{-2} = 2x$

OR

$y = v^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\dfrac{dv}{dx}$

$-\frac{1}{2}v^{-\frac{3}{2}}\dfrac{dv}{dx} + v^{-\frac{1}{2}} = -xv^{-\frac{3}{2}}$

$\dfrac{dv}{dx} - 2v = 2x$ $\quad \mu = e^{\int -2\,dx} = e^{-2x}$

$e^{-2x}\dfrac{dv}{dx} - 2e^{-2x}v = 2xe^{-2x}$ $\quad$ CHECK: $\dfrac{d}{dx}e^{-2x} = -2e^{-2x}$ ✓

$e^{-2x}v = \int 2xe^{-2x}\,dx + C$

$= -xe^{-2x} - \frac{1}{2}e^{-2x} + C$

u	dv
$2x$	e^{-2x}
2	$-\frac{1}{2}e^{-2x}$
0	$\frac{1}{4}e^{-2x}$

$v = -x - \frac{1}{2} + Ce^{2x}$

$y^{-2} = -x - \frac{1}{2} + Ce^{2x}$

NO PARTIAL CREDIT IF YOUR CHECKPOINT WAS MISSING, INCORRECT OR BOGUS

[b] $M = 2r \qquad N = 1 - r^2 \tan\theta$

$M_\theta = 0 \qquad N_r = -2r\tan\theta$

$$\frac{N_r - M_\theta}{M} = \frac{-2r\tan\theta}{2r} = -\tan\theta$$ FUNCTION OF ONLY θ

$$\mu = e^{\int -\tan\theta \, d\theta} = e^{\ln|\cos\theta|} = \cos\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$\underbrace{2r\cos\theta}_{M} \, dr + \underbrace{(\cos\theta - r^2\sin\theta)}_{N} \, d\theta = 0$$

$M_\theta = -2r\sin\theta = N_r$ EXACT ✓

$$f = \int 2r\cos\theta \, dr = r^2\cos\theta + C(\theta)$$

$$f_\theta = \cancel{-r^2\sin\theta} + C'(\theta) = \cos\theta \cancel{- r^2\sin\theta}$$

$$C(\theta) = \sin\theta$$

$$r^2\cos\theta + \sin\theta = C$$

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[c] $$\underbrace{t(x^2 - t^2)}_{M} \, dx + \underbrace{x(3t^2 - x^2)}_{N} \, dt = 0$$

$$M(kx, kt) = kt(k^2x^2 - k^2t^2) = k^3 t(x^2 - t^2) = t^3 M(x,t)$$

$$N(kx, kt) = kx(3k^2t^2 - k^2x^2) = k^3 x(3t^2 - x^2) = t^3 N(x,t)$$

HOMOGENEOUS

$x = vt \qquad dx = v\,dt + t\,dv$

$$t(v^2t^2 - t^2)(v\,dt + t\,dv) + vt(3t^2 - v^2t^2)\,dt = 0$$

$$(\cancel{v^3t^3} - vt^3 + 3vt^3 - \cancel{v^3t^3})\,dt + (v^2t^4 - t^4)\,dv = 0$$

$$2vt^3\,dt + (v^2 - 1)t^4\,dv = 0$$

$$\int \frac{1}{t}\,dt = \int \frac{1 - v^2}{2v}\,dv = \int \left(\frac{1}{2}\frac{1}{v} - \frac{1}{2}v\right)dv$$

$$\ln|t| = \frac{1}{2}\ln|v| - \frac{1}{4}v^2 + C$$

$$4\ln|t| = 2\ln\left|\frac{x}{t}\right| - \left(\frac{x}{t}\right)^2 + C$$

$$t^4 = \frac{Cx^2}{t^2} e^{-\frac{x^2}{t^2}}$$

$$t^6 e^{\frac{x^2}{t^2}} = Cx^2$$

OR

ALTERNATE SOLUTION FOR [b]

☆ USE ONLY 1 VERSION FOR GRADING

$$\frac{dr}{d\theta} - \frac{1}{2} r \tan\theta = -\frac{1}{2} r^{-1}$$ BERNOULLI

$$v = r^{1--1} = r^2$$

$$\frac{dv}{d\theta} = 2r \frac{dr}{d\theta}$$

$$2r \frac{dr}{d\theta} - r^2 \tan\theta = -1$$

OR

$$r = v^{\frac{1}{2}} \longrightarrow \frac{dr}{d\theta} = \frac{1}{2} v^{-\frac{1}{2}} \frac{dv}{d\theta}$$

$$\frac{1}{2} v^{-\frac{1}{2}} \frac{dv}{d\theta} - \frac{1}{2} v^{\frac{1}{2}} \tan\theta = -\frac{1}{2} v^{-\frac{1}{2}}$$

$$\frac{dv}{d\theta} - v \tan\theta = -1$$

$$\mu = e^{\int -\tan\theta \, d\theta} = e^{\ln|\cos\theta|} = \cos\theta$$

CHECK: $\frac{d}{d\theta} \cos\theta = -\sin\theta$ ✓

$$\cos\theta \frac{dv}{d\theta} - v \sin\theta = -\cos\theta$$

$$(\cos\theta) v = \int -\cos\theta \, d\theta + C$$

$$= -\sin\theta + C$$

$$v = -\tan\theta + C \sec\theta$$

$$r^2 = -\tan\theta + C \sec\theta$$

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ALTERNATE SOLUTION FOR [c]

OR ★ USE ONLY 1 VERSION FOR GRADING

$t = vx \qquad dt = v\,dx + x\,dv$

$$vx(x^2 - v^2x^2)dx + x(3v^2x^2 - x^2)(v\,dx + x\,dv) = 0$$

$$(\cancel{vx^3} - v^3x^3 + 3v^3x^3 - \cancel{vx^3})dx + (3v^2x^4 - x^4)dv = 0$$

$$2v^3x^3\,dx + x^4(3v^2 - 1)\,dv = 0$$

$$\int \frac{1}{x}\,dx = \int \frac{1 - 3v^2}{2v^3}\,dv = \int \left(\frac{1}{2}v^{-3} - \frac{3}{2}v^{-1}\right)dv$$

$$\ln|x| = -\frac{1}{4}v^{-2} - \frac{3}{2}\ln|v| + C$$

$$4\ln|x| = -\left(\frac{t}{x}\right)^{-2} - 6\ln\left|\frac{t}{x}\right| + C$$

$$x^4 = \frac{Cx^6}{t^6}e^{-\frac{x^2}{t^2}}$$

$$t^6 e^{\frac{x^2}{t^2}} = Cx^2$$

OR $(tx^2 - t^3)\,dx + (3t^2x - x^3)\,dt = 0$ 2ND ALTERNATE SOLUTION FOR [c]

☆ USE ONLY 1 VERSION FOR GRADING

GUESS $\mu = t^a x^b$

$$(\underbrace{t^{a+1}x^{b+2} - t^{a+3}x^b}_{M})\,dx + (\underbrace{3t^{a+2}x^{b+1} - t^a x^{b+3}}_{N})\,dt = 0$$

$$\boxed{\begin{aligned} M_t &= (a+1)t^a x^{b+2} - (a+3)t^{a+2}x^b \\ N_x &= 3(b+1)t^{a+1}x^b - (b+3)t^a x^{b+2} \end{aligned}}$$

$$a+1 = -(b+3)$$
$$-(a+3) = 3(b+1)$$
$$-2 = 2b \longrightarrow b = -1$$
$$a = -3$$

$$\mu = t^{-3}x^{-1}$$

$$(\underbrace{t^{-2}x - x^{-1}}_{M})\,dx + (\underbrace{3t^{-1} - t^{-3}x^2}_{N})\,dt = 0$$

$M_t = -2t^{-3}x = N_x$ EXACT ✓

$$f = \int (t^{-2}x - x^{-1})\,dx$$
$$= \tfrac{1}{2}t^{-2}x^2 - \ln|x| + C(t)$$

$$f_t = -\cancel{t^{-3}x^2} + C'(t) = 3t^{-1} - \cancel{t^{-3}x^2}$$
$$C(t) = 3\ln|t|$$

$$\tfrac{1}{2}t^{-2}x^2 - \ln|x| + 3\ln|t| = C$$

$$\frac{t^3}{x}e^{\frac{x^2}{2t^2}} = C$$

$$t^3 e^{\frac{x^2}{2t^2}} = Cx$$

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